MDC High School Mathematics Competition 2010
Miami Dade College

This is a paper and pencil examination.
Calculators may not be used.
There will be no bonus associated with finishing early.

Student name: _____

School name: _____

Solve.

1) $2x^2 - 4y^2 = 2$
   $5x^2 + 2y^2 = 53$
   - A) $(3,2),(2,3),(-3,-2),(-2,-3)$
   - B) $(3,2),(-3,2),(3,-2),(-3,-2)$
   - C) $(3,-2),(3,2)$
   - D) $(-3,-2),(-2,-3)$

2) $\sqrt{3x+1} = 3 + \sqrt{x-4}$
   - A) $\varnothing$
   - B) $5,8$
   - C) $-5,-8$
   - D) -1

3) $xy - y^2 = -60$
   $25xy - 10y^2 = 0$
   - A) $(-2,-5),(2,5)$
   - B) $\left(-\dfrac{4}{5},-50\right),\left(\dfrac{4}{5},50\right)$
   - C) $\left(-\dfrac{2}{5},-25\right),\left(\dfrac{2}{5},25\right)$
   - D) $(-4,-10),(4,10)$

4) $|x+5| - 4 = 11$
   - A) $12,10$
   - B) $-10,10$
   - C) $-2,10$
   - D) $-20,10$

5) $x^2 - xy + 3y^2 = 3$
   $2x^2 - 3xy + 4y^2 = 3$
   - A) $(1,1)$ and $(-1,-1)$
   - B) $(-1,1),(1,-1),(2,0),(0,-2)$
   - C) $(2,0)$ and $(0,-2)$
   - D) $(1,1),(-1,-1),(2,0),(0,-2)$

6) If a person puts 1 cent in a piggy bank on the first day, 2 cents on the second day, 3 cents on the third day, and so on, how much money will be in the bank after 50 days?

    A) $25.50   C) $0.50

    B) $6.38   D) $12.75

7) $x^2 + y^2 = 181$
$x + y = -19$

    A) $(9,-10),(10,-9)$ C) $(-9,10),(-10,9)$

    B) $(-9,-10),(-10,-9)$ D) $(9,10),(10,9)$

8) An auditorium has 20 rows with 10 seats in the first row, 12 in the second row, 14 in the third row, and so on. How many seats are in the auditorium?

    A) 580   C) 620

    B) 390   D) 400

9) $\cos^{-1} x + \cos^{-1} 2x = \cos^{-1} \dfrac{1}{2}$

    A) $x = 2$ C) $x = \pm \dfrac{3}{4}$

    B) $x = \dfrac{1}{2}$ D) $x = -5 \pm 3\sqrt{2}$

10) In how many distinguishable ways can the letters of the word WINDOW be arranged?

    A) 720   C) 36

    B) 360   D) 120

Evaluate.

11) $\cos \left(\sin^{-1} \dfrac{1}{4} \right)$

    A) $\dfrac{\sqrt{15}}{2}$ B) $\dfrac{2\sqrt{15}}{15}$ C) $\dfrac{4\sqrt{15}}{15}$ D) $\dfrac{\sqrt{15}}{4}$

12) $\sin\left(\tan^{-1}1 - \tan^{-1}0.8\right)$

    A) 82

    B) $\dfrac{1}{82}$

    C) $\sqrt{82}$

    D) $\dfrac{\sqrt{82}}{82}$

13) $\tan\left(2\tan^{-1}2\right)$

    A) $-\dfrac{2}{3}$

    B) $-\dfrac{4}{3}$

    C) $\dfrac{2}{3}$

    D) $\dfrac{4}{3}$

14) $\cos\left(\tan^{-1}x\right)$

    A) $\dfrac{1}{x}$

    B) x

    C) $\left(\sqrt{1+x^2}\right)x$

    D) $\dfrac{\sqrt{1+x^2}}{1+x^2}$

Evaluate exactly.

15) If $\cos\theta = \dfrac{5}{13}$ and $\cos\phi = \dfrac{3}{5}$, find $\cos(\theta+\phi)$.

    A) $\dfrac{-16}{65}$

    B) $\dfrac{56}{65}$

    C) $\dfrac{63}{65}$

    D) $-\dfrac{33}{65}$

16) $\cos\dfrac{5\pi}{12}$

    A) $\dfrac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}$

    B) $-\sqrt{2}\left(\sqrt{3}-1\right)$

    C) $\sqrt{2}\left(\sqrt{3}-1\right)$

    D) $-\dfrac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}$

Solve, finding all solutions in $[0, 2\pi)$.

17) $\ln|\sin x| = 0$

     A) $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$ B) $\pi, \dfrac{5\pi}{4}$ C) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$ D) $\dfrac{\pi}{3}, \dfrac{2\pi}{3}$

18) $4\sin^2 x = 4\cos x + 1$

     A) $x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ C) $x = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$

     B) $x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ D) No solution

19) $\cos\left(\dfrac{\pi}{2} + x\right) + \sin(x - \pi) = 1$

     A) $\dfrac{\pi}{6}, \dfrac{7\pi}{6}$ C) $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

     B) $\dfrac{\pi}{4}, \dfrac{3\pi}{4}$ D) $\dfrac{7\pi}{6}$

20) $2\sin x \cos x + \cos x = 0$

     A) No solution

                                         C) $x = \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}, \dfrac{11\pi}{6}$

     B) $x = \dfrac{\pi}{2}, \dfrac{7\pi}{6}, \dfrac{3\pi}{2}, \dfrac{11\pi}{6}$ D) $x = \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}, 1$

21) $2\sin x \cos x - 2\sin x + \cos x = 1$
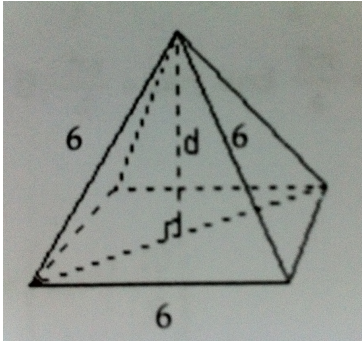
     A) $x = 0, \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$ C) No solution

     B) $x = 0, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ D) $x = 0, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$

Find d in simplest radical form.

22)

Right square pyramid with equilateral triangular faces.

   A) $6\sqrt{2}$ B) $3\sqrt{3}$ C) $6\sqrt{3}$ D) $3\sqrt{3}$

23) $\ln(3x-1)=\ln 1-\ln(x-1)$

   A) $1,\dfrac{1}{3}$ B) $0,\dfrac{4}{3}$ C) $\dfrac{4}{3}$ D) $\varnothing$

24) $\log(3+x)-\log(x-4)=\log 4$

   A) $-\dfrac{19}{3}$ C) $\dfrac{5}{2}$

   B) $\varnothing$ D) $\dfrac{19}{3}$

Solve the following system.

25) $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=\dfrac{5}{6}$

   $\dfrac{1}{x}-\dfrac{1}{y}-\dfrac{6}{z}=-\dfrac{1}{2}$

   $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=\dfrac{5}{6}$

   A) $\left(1,-\dfrac{1}{2},\dfrac{1}{3}\right)$ C) $(1,-2,-3)$

   B) $(1,-2,3)$ D) $(-1,2,3)$

Solve the exponential equation.

26) $16 = 2^{7x} \cdot 4^{x^2}$

A) $4, -\dfrac{1}{2}$

B) $-4, -2$

C) $4, 2$

D) $-4, \dfrac{1}{2}$

Solve, finding all solutions.

27) $\sec x = \sqrt{2}$ (Express your answer in radians.)

A) $2k\pi$ and $\pi + 2k\pi$, where k is any integer

B) $\dfrac{\pi}{4} + 2k\pi$ and $\dfrac{7\pi}{4} + 2k\pi$, where k is any integer

C) $\dfrac{\pi}{2} + 2k\pi$ and $\dfrac{3\pi}{2} + 2k\pi$, where k is any integer

D) $\dfrac{3\pi}{4} + 2k\pi$ and $\dfrac{5\pi}{4} + 2k\pi$, where k is any integer

Find.

28) $\tan\left(\sin^{-1} \dfrac{10}{y} \right)$

A) $\dfrac{y^2 + 100}{10}$

B) $\dfrac{10}{y^2 + 100}$

C) $\dfrac{10}{\sqrt{y^2 - 100}}$

D) $\dfrac{10}{\sqrt{y^2 + 100}}$

Factor and simplify.

29) $\dfrac{\csc x \left(\sin^2 x + \cos^2 x \tan x \right)}{\sin x + \cos x}$
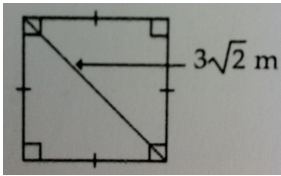
A) $\sin x + \cos x$

B) $\csc x \tan x$

C) 1

D) $\csc^2 x + \tan^2 x$

Find the perimeter.

30)

A) 12 m

B) $12\sqrt{2}$ m

C) 6 m

D) 9 m

Solve the problem.

31) The sum of a student's three scores is 228. If the first is 17 points more than the second, and the sum of the first two is 15 more than twice the third, what is the first score?

A) 87

B) 54

C) 71

D) 70

Simplify.

32) $\cos(2x+5y)+\cos(2x-5y)$
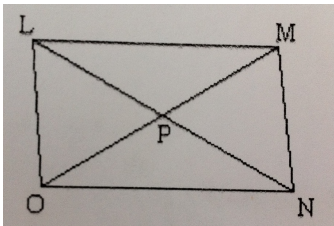
A) $2\sin 2x \sin 5y$

B) $\sin 4x \cos 10y$

C) $2\cos 2x \cos 5y$

D) $2\sin 2x \cos 5y$

Find the measures of the indicated sides, angles, or diagonals of the given parallelogram.

33) $\overline{PM}=32$ and $\overline{LP}=46$. Find the length or each diagonal.

A) $\overline{LN}=46$, $\overline{MO}=32$

B) $\overline{LN}=92$, $\overline{MO}=64$

C) $\overline{LN}=\overline{MO}=78$

D) $\overline{LN}=64$, $\overline{MO}=92$

Simplify the expression.

34) $\dfrac{2\sin^5 x}{\cos^2 x} \cdot \left(\dfrac{\cos x}{2\sin x}\right)^2$

    A) $4\sin^3 x$

    B) $\dfrac{1}{4}\sin^3 x$

    C) $\dfrac{1}{2}\sin^3 x$

    D) $\dfrac{1}{2}\sin^2 x$

Find the sum, if it exists.

35) $-29 - \dfrac{58}{9} - \dfrac{116}{81} - \dfrac{232}{729} - \ldots$

    A) $-\dfrac{58}{7}$

    B) $-\dfrac{261}{7}$

    C) $\dfrac{9}{7}$

    D) Does not exist

Solve, finding all solutions in $[0,2\pi)$

36) $5.4\cos x + 2.7\sqrt{3} = 0$

    A) $\dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

    B) $\dfrac{\pi}{6}, \dfrac{7\pi}{6}$

    C) $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

    D) $\dfrac{5\pi}{6}, \dfrac{7\pi}{6}$